Energy-Distortion Tradeoff with Multiple Sources and Feedback

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***Abstract*—The energy-distortion tradeoff for lossy transmission of sources over multi-user networks is studied. The *energy-distortion function* $E(D)$ is defined as the minimum energy required to transmit a source to the receiver within the target distortion D, when there is no restriction on the number of channel uses per source sample. For point-to-point channels, $E(D)$ is shown to be equal to the product of the minimum energy per bit $E_{b\min}$ and the rate–distortion function $R(D)$, indicating the optimality of source-channel separation in this setting. It is shown that the optimal $E(D)$ can also be achieved by the Schalkwijk–Kailath (SK) scheme, as well as separate coding, in the presence of perfect channel output feedback.**

Then, it is shown that the optimality of separation in terms of $E(D)$ does not extend to multi-user networks. The scenario with two encoders observing correlated Gaussian sources in which the encoders communicate to the receiver over a Gaussian multiple-access channel (MAC) with perfect channel output feedback is studied. First a lower bound on $E(D)$ is provided and compared against two upper bounds achievable by separation and an uncoded SK type scheme, respectively. Even though neither of these achievable schemes meets the lower bound in general, it is shown that their energy requirements lie within a constant gap of $E(D)$ in the low distortion regime, for which the energy requirement grows unbounded. It is shown that the SK based scheme outperforms the separation based scheme in certain scenarios, which establishes the sub-optimality of separation in this multi-user setting.

I. Introduction

The fundamental problem in communications is to transmit a message from a source terminal to a destination over a noisy channel such that the destination can reconstruct the source message with the highest fidelity. In general, we can associate a cost for using the channel and also define the fidelity of the reconstruction by a distortion function. Naturally, there is a tradeoff between the available budget for transmission and the achievable distortion at the destination. In classical models, it is assumed that the system designer is given a certain average budget per each use of the channel as well as a fixed bandwidth ratio that specifies the number of channel uses per source sample. Then the problem is to find the minimum budget (per channel use) required to achieve a target distortion requirement for the fixed bandwidth ratio. The solution characterizes the *power-distortion tradeoff* for the given system at the fixed bandwidth ratio. In this work, we introduce an *energy-distortion tradeoff*. The 'energy' refers to the total cost of using the communication channel per source sample. Thus, rather than constraining the cost of each channel use for a fixed bandwidth ratio, we constrain the total budget used over the channel. We capture the corresponding tradeoff by the fundamental information-theoretic function $E(D)$ which is the minimum energy required per source sample to achieve an average distortion D, for a large number of source samples. We note that, in this model, no restriction has been placed on the bandwidth ratio, thus allowing us to maximize the energy efficiency over unlimited bandwidth.

A potential application for our model is sensor networks, in which various physical phenomena observed at the sensor nodes are to be reconstructed at the fusion center. Ultra-wideband has been considered as a viable communication strategy for sensor networks because of several benefits including high performance in the low power regime [3]. In our model, by removing the bandwidth ratio constraint, we basically identify the fundamental performance limits for the energy-distortion tradeoff in the wideband limit.

In a point-to-point communication system, separate source and channel coding is known to be optimal in terms of the power-distortion tradeoff. Naturally, the separation optimality applies to the energy-distortion tradeoff as well: the optimal E(D) is achieved by lossy compression (at rate R(D) per source sample) followed by channel encoding in the most energy efficient manner, i.e., by operating in the wideband regime such that the transmitter uses minimum energy per bit $E_{b\min}$. We also consider the scenario in which perfect channel output feedback is available at the transmitter. We show that, similarly to the power-distortion tradeoff, energy-distortion tradeoff also remains the same despite the additional feedback link.

It is yet another well-known fact that the optimality of source-channel separation does not extend to multi-user scenarios other than in a number of special cases [1], [4]. We focus on the case with multiple sources, in which two sensors observing correlated sources want to transmit their observation to a fusion center over a multiple access channel (MAC). In particular, we consider two encoders/transmitters observing Gaussian sources which are correlated. The communication channel from the transmitters to the receiver is assumed to be an additive white Gaussian noise (AWGN) channel. Moreover, we assume the availability of perfect channel output feedback at both transmitters. The power-distortion tradeoff for this model is studied in [8] in the case of matching source and channel bandwidths. We are interested in obtaining the minimum energy requirement for reconstructing both the sources at the receiver within a target distortion without any restrictions on the source and channel bandwidths. For the sake of simplicity of analysis we restrict our attention to the case of symmetric sources, energies, channel gains and target

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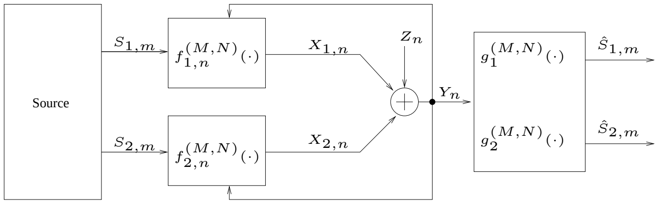


Fig. 1: Bivariate Gaussian source model with perfect channel output feedback.

distortions.

The rest of the paper is organized as follows. After introducing the general system model for two users in Section II, we present the results for a point-to-point system in Section III. For the two-user model, we provide a converse bound similarly to [8] by simply extending their analysis to the bandwidth mismatch case in Section IV. For achievability, in Section V we analyze a separate source and channel coding scheme, in which the feedback information is used only to improve the channel transmission rates of the transmitters. Then in Section VI we study an uncoded transmission scheme motivated by the analog transmission technique introduced by Schalkwijk and Kailath in [6] and applied to the MAC with feedback by Ozarow in [10]. We characterize the energy-distortion tradeoff for both of these schemes. Section VII is dedicated to the comparison of these bounds for some chosen values of correlation coefficient; and finally Section VIII concludes the paper.

II. System Model

We consider a MAC with two transmitters in which perfect channel output at the receiver end is available as feedback to both of the transmitters. The source S_i^M at transmitter i is an M-length random vector of independent and identically distributed (i.i.d.) real-valued Gaussian random variables with zero means and variances $\sigma_{S_i}^2$, i.e., $S_i \sim \mathcal{N}(0, \sigma_{S_i}^2)$ for $i = 1, 2$. We assume that the sources are correlated , and the joint bivariate Gaussian distribution has the covariance matrix

$$\begin{bmatrix} \sigma_{S_1}^2 & \rho\sigma_{S_1}\sigma_{S_2} \\ \rho\sigma_{S_1}\sigma_{S_2} & \sigma_{S_2}^2 \end{bmatrix}, \tag{1}$$

where $-1 \leq \rho \leq 1$ is the correlation coefficient. Without loss of generality, ρ can be taken to lie within $[0, 1]$ as otherwise, if the sources are negatively correlated, we can replace S_2 by $-S_2$ to obtain a non-negative correlation coefficient.

Transmitters encode their observations and transmit them over a MAC. Denoting the input sequence at transmitter i as X_i^N, and the corresponding channel output vector as Y^N, the channel is characterized by

$$Y_n = X_{1,n} + X_{2,n} + Z_n \qquad \text{for } n = 1, \ldots, N, \tag{2}$$

where $Z^N = (Z_1, ..., Z_N)$ is the vector of i.i.d. $\mathcal{N}(0, \sigma_Z^2)$ channel noise variables. In this work, we focus on the symmetric scenario in which the source statistics are the same, i.e., $\sigma_{S_1}^2 = \sigma_{S_2}^2 \triangleq \sigma_S^2$. See Fig. 1 for an illustration of the system model. We denote this system the $(\sigma_S^2, \sigma_Z^2, \rho)$ network.

Moreover, we assume that perfect causal channel output feedback is available at both of the transmitters, hence the encoding function at each transmitter depends not only on the source vector but also on the previous channel outputs. Considering block encoding from an M-length source vector to an N-length channel vector, the encoder at transmitter i is described by a sequence of encoding functions $f_{i,n}^{(M,N)} : \mathbb{R}^M \times \mathbb{R}^{n-1} \to \mathbb{R}$ where $X_{i,n} = f_{i,n}^{(M,N)}(S_i^M, Y^{n-1})$, for $i = 1, 2$ and $n = 1, \ldots, N$. The decoder is described by a pair of decoding functions $g_i^{(M,N)} : \mathbb{R}^N \to \mathbb{R}^M$ where $\hat{S}_i^M = g_i^{(M,N)}(Y^N)$, for $i = 1, 2$.

Definition 2.1: For a $(\sigma_S^2, \sigma_Z^2, \rho)$ network, an energy-distortion pair (E, D) is said to be *achievable* if there exists a sequence (over M) of encoding functions

$$\{f_{1,n}^{(M,N)}\}_{n=1}^N \text{ and } \{f_{2,n}^{(M,N)}\}_{n=1}^N$$

satisfying the energy constraint

$$\mathbb{E}\left[\sum_{n=1}^N X_{i,n}^2\right] \leq ME \qquad \text{for } i = 1, 2, \tag{3}$$

and a sequence of decoding functions

$$g_1^{(M,N)} \text{ and } g_2^{(M,N)}$$

such that the corresponding distortion sequence satisfies

$$\limsup_{M\to\infty} \frac{1}{M} \sum_{m=1}^M \mathbb{E}\left[(S_{i,m} - \hat{S}_{i,m})^2\right] \leq D, \qquad \text{for } i = 1, 2.$$

Definition 2.2: We define the *energy-distortion function* for the $(\sigma_S^2, \sigma_Z^2, \rho)$ network as

$$E(D) \triangleq \inf\{E \geq 0 : (E, D) \text{ is achievable}\}. \tag{4}$$

Note that we do not impose any constraints on the bandwidth ratio, and hence, it is possible to transmit as many channel symbols per source observation as desired as long as the total energy constraint is satisfied. Our goal in this paper is to determine $E(D)$ for a $(\sigma_S^2, \sigma_Z^2, \rho)$ network.

III. Single Source Scenario

We first treat the single user scenario without feedback, in which a single Gaussian source S is to be transmitted over a point-to-point AWGN channel with noise variance σ_Z^2. To present our result we first define $E_{b\min}$ as the *minimum energy per bit* [12] for the underlying communication channel, and $R(D)$ as the rate-distortion function for the given source, that is, the minimum rate (bits per source sample) of encoding S required to achieve an average distortion D.

Lemma 3.1: For the single source scenario, we have

$$E(D) = E_{b\min} \times R(D), \tag{5}$$

which can be achieved by separate source and channel coding[1].

Lemma 3.1 holds in considerable generality, including stationary sources and channels with memory. For the AWGN channel and the Gaussian source, we have

$$E_{b\min} = 2\sigma_Z^2 \log_e 2, \tag{6}$$

and

$$R(D) = \frac{1}{2}\log_2^+\left(\frac{\sigma_S^2}{D}\right), \tag{7}$$

[1] Proofs are not included due to space limitations.

where $\log^+(x) = \log(x)$ if $x \geq 1$ and 0 otherwise. Therefore, we have

$$E(D) = \sigma_Z^2 \log_e^+ \left(\frac{\sigma_S^2}{D}\right). \quad (8)$$

We remark next that the optimality of separation in terms of the power-distortion tradeoff holds even in the presence of perfect channel output feedback, and moreover, the tradeoff remains the same since the point-to-point channel capacity is not improved by feedback. However, in the case of an AWGN channel, perfect channel output feedback can be used to achieve the optimal tradeoff by the simpler uncoded Schalkwijk-Kailath (SK) scheme [6]. In the SK scheme for a point-to-point channel, the transmitter transmits a scaled version of the estimation error at the receiver in an uncoded manner in each step; hence, it achieves the channel capacity without incurring any coding delay. The SK scheme can be adapted to joint source–channel coding, i.e., no compression of the source, and it achieves the optimal power-distortion tradeoff in the point-to-point setting for any fixed bandwidth ratio [11]. So, the SK scheme extends the optimality of uncoded transmission in point-to-point systems to the bandwidth mismatch case.

It is possible to prove that similar arguments hold for the energy-distortion tradeoff as well. Lemma 3.1 can be extended to show that (5) holds even when perfect channel feedback is available at the transmitters. Since feedback does not change $E_{b\min}$, $E(D)$ also remains the same, and this can also be achieved by the SK scheme.

For the rest of the paper, we study the scenario with two correlated sources. We provide lower and upper bounds on $E(D)$.

IV. Lower Bound on $E(D)$

Here, we present a lower bound on $E(D)$ for the $(\sigma_S^2, \sigma_Z^2, \rho)$ network. First assume perfect cooperation between the two encoders. This reduces the model to that of a single vector source and a 2×1 multiple input - single output point-to-point channel with individual distortion requirements on the components of the vector source. We can directly apply Lemma 3.1 with

$$E_{b\min} = \sigma_Z^2 \log_e 2, \quad (9)$$

which is the minimum energy per bit for this channel. The rate–distortion function under individual distortion constraints is given by [14]

$$R(D) = \begin{cases} \frac{1}{2}\log_2^+\left(\frac{\sigma_S^4(1-\rho^2)}{D^2}\right) & \text{if } 0 \leq D \leq \sigma_S^2(1-\rho) \\ \frac{1}{2}\log_2^+\left(\frac{\sigma_S^2(1+\rho)}{2D-(1-\rho)\sigma_S^2}\right) & \text{if } \sigma_S^2(1-\rho) < D \leq \sigma_S^2. \end{cases} \quad (10)$$

Theorem 4.1: In a $(\sigma_S^2, \sigma_Z^2, \rho)$ network, $E(D)$ is lower bounded by $E_{\text{lb1}}(D)$ where

$$E_{\text{lb1}}(D) = \begin{cases} \frac{\sigma_Z^2}{4}\log_e^+\left(\frac{\sigma_S^4(1-\rho^2)}{D^2}\right) \\ \qquad \text{if } 0 \leq D \leq \sigma_S^2(1-\rho) \\ \frac{\sigma_Z^2}{4}\log_e^+\left(\frac{\sigma_S^2(1+\rho)}{2D-(1-\rho)\sigma_S^2}\right) \\ \qquad \text{if } \sigma_S^2(1-\rho) < D \leq \sigma_S^2. \end{cases} \quad (11)$$

This lower bound can be tightened by following the arguments in [7, Section 3]. This bound considers separate channel encoding at the transmitters, and bounds the amount of correlation that can be created among the channel codewords by using the available correlation among the source vectors. This allows us to improve the lower bound by better accounting for the limited beamforming gain over the channel.

We define the conditional rate-distortion functions $R_{S_1|S_2}(D)$ and $R_{S_2|S_1}(D)$ as the minimum rate required to achieve a distortion of D for one source when the other source is available at both the encoder and the decoder. It can be shown that

$$R_{S_1|S_2}(D) = R_{S_2|S_1}(D) = \frac{1}{2}\log_2^+\left(\frac{\sigma_S^2(1-\rho^2)}{D}\right). \quad (12)$$

Theorem 4.2: In a $(\sigma_S^2, \sigma_Z^2, \rho)$ network, $E(D)$ is lower bounded by $E_{\text{lb2}}(D)$ as given in (13).

Since $E_{\text{lb1}}(D) \leq E_{\text{lb2}}(D)$, we restrict our attention to the tighter bound $E_{\text{lb2}}(D)$ for the rest of the paper.

V. Separate Source and Channel Coding

Apart from the practical motivation due to the modularity it provides, separate source and channel coding is also motivated by its theoretical optimality in the point-to-point scenario. Almost all practical communication systems operate based on separate source and channel coding. In this section, we outline a separation based scheme and analyze its energy-distortion tradeoff.

In separate source and channel coding, the encoders first quantize their sources at identical rates through distributed compression (see [9] or [13]) and then transmit the quantized information bits over the MAC with perfect feedback, operating on the boundary of the capacity region for this channel which was characterized by Ozarow in [10].

The rate-distortion function $R_{\text{sep}}(D)$ to achieve symmetric distortion D for each source is given as [9], [13]

$$R_{\text{sep}}(D) = \max\left\{\frac{1}{2}\log_2^+\left(\frac{\sigma_S^2(1-\rho^2)}{2D}\left(1+\sqrt{1+\frac{4D\rho^2}{\sigma_S^2(1-\rho^2)^2}}\right)\right), \frac{1}{4}\log_2^+\left(\frac{\sigma_S^4(1-\rho^2)}{2D^2}\left(1+\sqrt{1+\frac{4D^2\rho^2}{\sigma_S^4(1-\rho^2)^2}}\right)\right)\right\}. \quad (14)$$

$$E_{\text{lb2}}(D) = \begin{cases} \min_{0\leq\hat{\rho}\leq 1}\max\left\{\frac{\sigma_Z^2}{(1-\hat{\rho}^2)}\log_e^+\left(\frac{\sigma_S^2(1-\rho^2)}{D}\right), \frac{\sigma_Z^2}{2(1+\hat{\rho})}\log_e^+\left(\frac{\sigma_S^4(1-\rho^2)}{D^2}\right)\right\} & \text{if } 0 \leq D \leq \sigma_S^2(1-\rho) \\ \min_{0\leq\hat{\rho}\leq 1}\max\left\{\frac{\sigma_Z^2}{(1-\hat{\rho}^2)}\log_e^+\left(\frac{\sigma_S^2(1-\rho^2)}{D}\right), \frac{\sigma_Z^2}{2(1+\hat{\rho})}\log_e^+\left(\frac{\sigma_S^2(1+\rho)}{2D-(1-\rho)\sigma_S^2}\right)\right\} & \text{if } \sigma_S^2(1-\rho) < D \leq \sigma_S^2 \end{cases} \quad (13)$$

However, in the case of energy-distortion tradeoff, it is sufficient for each transmitter to transmit $R_{\text{sep}}(D)$ bits to the receiver in a separate, orthogonal band, i.e., without interfering with the other transmitter. Note that the feedback signal is not used in this scheme. This is because, despite the fact that feedback enlarges the capacity region of a MAC under average power constraints at the users, it does not improve the minimum energy per bit which is achieved by orthogonal transmissions. The achievable energy-distortion tradeoff of the separation scheme $E_{\text{sep}}(D)$ is given in the next theorem.

Theorem 5.1: In a $(\sigma_S^2, \sigma_Z^2, \rho)$ network, $E(D)$ is upper bounded by $E_{\text{sep}}(D)$, where

$$E_{\text{sep}}(D) = \max\left\{\sigma_Z^2 \log_e^+\left(\frac{\sigma_S^2(1-\rho^2)}{2D}\left(1+\sqrt{1+\frac{4D\rho^2}{\sigma_S^2(1-\rho^2)^2}}\right)\right), \frac{\sigma_Z^2}{2}\log_e^+\left(\frac{\sigma_S^4(1-\rho^2)}{2D^2}\left(1+\sqrt{1+\frac{4D^2\rho^2}{\sigma_S^4(1-\rho^2)^2}}\right)\right)\right\}. \quad (15)$$

Remark 5.1: For $\rho = 0$, it can be easily checked that the lower bound $E_{\text{lb2}}(D)$ and upper bound $E_{\text{sep}}(D)$ match. This is expected since separation is optimal for independent messages. For $\rho = 1$, $E_{\text{sep}}(D)$ is twice the lower bound $E_{\text{lb2}}(D)$. From this extreme case of identical sources, we can conclude that one of the reasons for the suboptimal performance of separate source and channel coding is the independence of channel codewords, i.e., no beamforming gain can be exploited in separation despite the available correlation among the sources.

While separation is not optimal in general, the next proposition states that it has only a finite energy gap with the optimal performance even as $E(D)$ diverges to infinity in the low distortion regime. Hence, separation can be a viable alternative for real-world applications when the source correlation is low. Note that the gap between $E_{\text{sep}}(D)$ and $E_{\text{lb2}}(D)$ diverges as $\rho \to 1$.

Proposition 5.2: In a $(\sigma_S^2, \sigma_Z^2, \rho)$ network, the following holds:

$$\lim_{D\to 0} E_{\text{sep}}(D) - E_{\text{lb2}}(D) = \frac{\sigma_Z^2}{2}\log_e\left(\frac{1}{1-\rho^2}\right), \quad (16)$$

whereas

$$\lim_{D\to 0} E_{\text{sep}}(D) = \infty \quad \text{and} \quad \lim_{D\to 0} E_{\text{lb2}}(D) = \infty. \quad (17)$$

VI. Uncoded Transmission

While separation is optimal for point-to-point systems and for a MAC with independent sources, this is not the case when the sources are correlated. In this section, we describe an alternative achievability scheme with uncoded transmission based on the Schalkwijk-Kailath (SK) scheme [6]. The advantage of uncoded transmission in multi-user scenarios has been shown previously. In Gaussian sensor networks, when the source and channel bandwidths match, it is known that the uncoded transmission is exactly optimal [2]. In [5], we showed that an SK based uncoded transmission scheme has better energy-distortion performance than the separation based scheme in certain cases. In fact, in the case of a MAC with perfect feedback the channel coding part of the separation based transmission of correlated sources inherently uses uncoded transmission since the achievability of the capacity region is based on the SK scheme [10]. Here, we basically extend the transmission scheme of [10] to the scenario of correlated sources.

The basic idea is similar to the SK scheme for a point-to-point channel. In each step, each transmitter calculates the 'error' for its own source, i.e., the difference between the estimate at the receiver and the actual source realization at the transmitter. These errors are then scaled and transmitted simultaneously by both transmitters over the MAC. The power of these transmissions at every channel use is taken to be fixed and very small (approaching zero). Based on the received signals, the receiver updates its estimates for both of the sources. The scheme is terminated once the target distortions for both sources are achieved at the receiver. Note that, as mentioned above, the achievability part (channel coding) of the separation scheme uses a similar uncoded scheme at its core. The main difference of the scheme proposed here from the separation based scheme is that now we eliminate the 'quantization' step and deal directly with the source realizations at the transmitters.

Theorem 6.1: In a $(\sigma_S^2, 1, \rho)$ network, $E(D)$ is upper bounded by $E_{\text{u}}(D)$, where

$$E_{\text{u}}(D) = \begin{cases} \frac{1}{4}\log_e\left(\frac{(1+\rho)\sigma_S^2}{2D-(1-\rho)\sigma_S^2}\right) + \frac{1}{2}\left(\frac{D}{2D-(1-\rho)\sigma_S^2} - \frac{1}{1+\rho}\right) & \text{if } D \geq \sigma_S^2(1-\rho) \\ \frac{1}{4}\log_e\left(\frac{1+\rho}{1-\rho}\right) + \frac{1}{2}\left(\frac{\rho}{1+\rho}\right) + \log_e\left(\frac{(1-\rho)\sigma_S^2}{D}\right) & \text{if } 0 \leq D \leq \sigma_S^2(1-\rho). \end{cases} \quad (18)$$

Similarly to Proposition 5.2, we can bound the energy gap between the energy requirement of the uncoded scheme and the optimal one.

Proposition 6.2: In a $(\sigma_S^2, 1, \rho)$ network, the following holds:

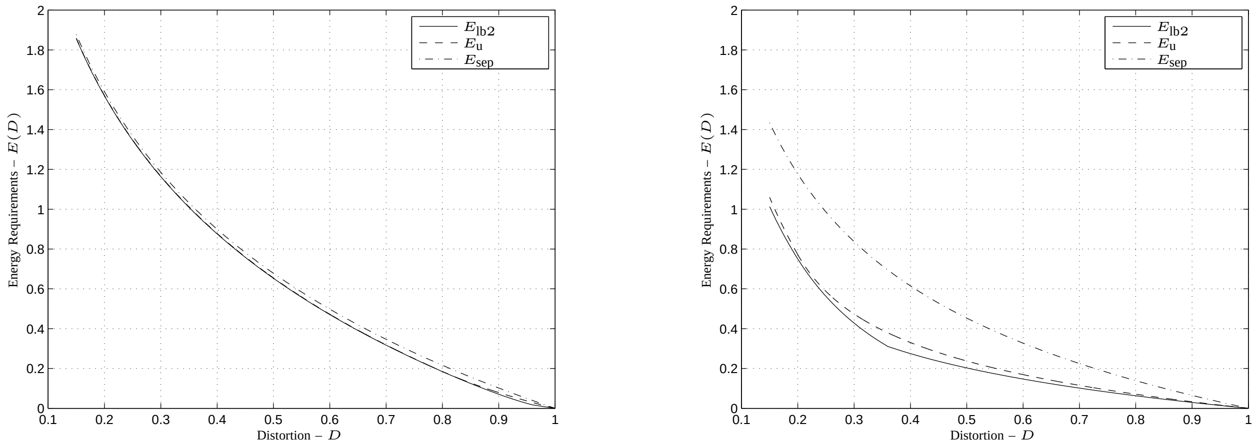
$$\lim_{D\to 0} E_{\text{u}}(D) - E_{\text{lb2}}(D) = \frac{\rho}{2(1+\rho)} - \frac{1}{4}\log_e\left((1-\rho)(1+\rho)^3\right), \quad (19)$$

whereas

$$\lim_{D\to 0} E_{\text{u}}(D) = \infty \quad \text{and} \quad \lim_{D\to 0} E_{\text{lb2}}(D) = \infty. \quad (20)$$

VII. Numerical Results and Discussions

In Figs. (2a) and (2b), we plot the lower and upper bounds on $E(D)$ for $\sigma_S^2 = \sigma_Z^2 = 1$ and ρ equal to 0.2 and 0.8 respectively. We observe in both cases that the uncoded transmission scheme performs better than separation at all distortion requirements plotted in the figures. For $\rho = 0.2$, all the bounds are close to each other. However, the gap between the performance of the uncoded transmission scheme and the lower bound is almost indistinguishable except at higher values of distortion. The curves are more separated when $\rho = 0.8$. In this case, the uncoded transmission scheme has a clear advantage over the separation based scheme. Note that the finite energy gap identified for the uncoded transmission scheme in Prop. 6.2 is smaller than the one for the separation scheme in Prop. 5.2; however, this does not directly lead to the superiority of the uncoded scheme.



(a) $\rho = 0.2$

(b) $\rho = 0.8$

Fig. 2: $E(D)$ bounds for a $(1, 1, \rho)$ network, for $\rho = 0.2$ and $\rho = 0.8$.

Moreover, the uncoded transmission scheme is exactly optimal when $\rho = 0$ or $\rho = 1$. Furthermore, when $\rho = 0$, the separation based scheme is also optimal though it has exactly twice the energy consumption of the lower bound when $\rho = 1$. This also implies that the energy consumption of the uncoded transmission scheme is half as much as that of the separation based scheme when $\rho = 1$.

VIII. Conclusions

We have studied the fundamental energy-distortion tradeoff function $E(D)$ in networks with one or two Gaussian sources and a single receiver when there is no constraint on the available channel bandwidth per source sample. We have considered the scenario in which the perfect channel output is available at the transmitters causally. Using separation, $E(D)$ has been established for the point-to-point scenario. For the case of two sources, we have first provided a lower bound on $E(D)$. This lower bound represents the absolute minimum energy (in Joules) that is required to reconstruct the sources within the target distortion at the receiver, regardless of the communication/reconstruction strategies used in the system. The lower bound is tight when the sources are independent. Besides the lower bound, we have also studied two different upper bounds based on separation and uncoded transmission, respectively. Simulation results suggest that uncoded transmission can beat the separation based scheme in many situations, proving the suboptimality of separation in this model. This also illustrates that uncoded transmission might be attractive in multi-user systems from an energy efficiency perspective, extending a similar observation in [8] to the wideband regime. Moreover, we have shown that both the separate and the uncoded schemes require at most a finite amount of extra energy than the minimal one, even in the limit of zero distortion, in which case the energy requirement diverges.

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